Problem of the Week
Problem D and Solution
Sum to 1000

Problem
The number 1000 can be written as the sum of 16 consecutive whole numbers:

\[ 1000 = 55 + 56 + 57 + 58 + 59 + 60 + 61 + 62 + 63 + 64 + 65 + 66 + 67 + 68 + 69 + 70. \]

Using at least two numbers, what is the minimum number of consecutive whole numbers needed to sum to exactly 1000?

Solution
In this solution we will examine possible cases until we discover the first one that works.

1. Can 1000 be written using 2 consecutive whole numbers?
   Let \( n, n + 1 \) represent the two numbers. Then,
   \[
   \begin{align*}
   n + n + 1 &= 1000 \\
   2n + 1 &= 1000 \\
   2n &= 999 \\
   n &= 499.5
   \end{align*}
   \]
   Since \( n \) is not a whole number, it is not possible to write 1000 using two consecutive whole numbers.

2. Can 1000 be written using 3 consecutive whole numbers?
   Let \( n, n + 1, n + 2 \) represent the three numbers. Then,
   \[
   \begin{align*}
   n + n + 1 + n + 2 &= 1000 \\
   3n + 3 &= 1000 \\
   3n &= 997 \\
   n &= 332.3
   \end{align*}
   \]
   Since \( n \) is not a whole number, it is not possible to write 1000 using three consecutive whole numbers. (Refer to the note following the solution for an alternate way to define the three consecutive whole numbers.)

The solution continues on the next page.
3. Can 1000 be written using 4 consecutive whole numbers?

Let \( n, n+1, n+2, n+3 \) represent the four numbers. Then,
\[
\begin{align*}
    n + n + 1 + n + 2 + n + 3 &= 1000 \\
    4n + 6 &= 1000 \\
    4n &= 994 \\
    n &= 248.5
\end{align*}
\]

Since \( n \) is not a whole number, it is not possible to write 1000 using four consecutive whole numbers.

4. Can 1000 be written using 5 consecutive whole numbers?

Let \( n, n+1, n+2, n+3, n+4 \) represent the five numbers. Then,
\[
\begin{align*}
    n + n + 1 + n + 2 + n + 3 + n + 4 &= 1000 \\
    5n + 10 &= 1000 \\
    5n &= 990 \\
    n &= 198
\end{align*}
\]

Since \( n \) is a whole number, it is possible to write 1000 using five consecutive whole numbers. That is, \( 1000 = 198 + 199 + 200 + 201 + 202 \).

Since we have checked all possible numbers of consecutive whole numbers below five and none of them worked, the minimum number of consecutive whole numbers required to produce a sum of 1000 is five.

**Note:**

In the second case, when we checked to see if 1000 could be written as the sum of three consecutive whole numbers. We could have proceeded as follows:

Let \( a - 1, a, a + 1 \) represent the three consecutive whole numbers. Then,
\[
\begin{align*}
    a - 1 + a + a + 1 &= 1000 \\
    3a &= 1000 \\
    a &= \frac{1000}{3} = 333.3
\end{align*}
\]

Since \( a \) is not a whole number, it is not possible to write 1000 using three consecutive whole numbers.

This “idea” is useful when we are finding the sum of an odd number of consecutive integers. We could also have applied the same idea to the fourth case by using \( a - 2, a - 1, a, a + 1, a + 2 \) to represent the five consecutive positive integers.